Spectral Super Elements for Beams under Moving Loads for Arbitrary Boundary Conditions

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Introduction
Augmented finite beam elements for the calculation of structures with one direction of primary extend and arbitrary cross sections have been developed at the Chair of Structural Mechanics [1]. A separation of variables in the continuum of the beam-shaped structure is carried out, which separates the longitudinal dependency form the cross-sectional dependency of the deflections. The cross sectional dependency is calculated in a pre-processing step in the form of unit deflection shapes. Different methods for that calculation have been proposed. The unit deflection shapes can be understood as new degrees of freedom in the scope of a beam theory. In contrast to classical beam theories (e.g. Euler-Bernoulli or Timoshenko) distortions of the cross-section (in-plane as well as out-of-plane) are enabled thereby.

Fig. 1: Separation of variables; cross-sectional displacement at each beam node as a linear combination of unit deflection shapes $\psi_i$ and their contribution factors $b_{a,i}$
The longitudinal variables are the contribution factors of each unit deflection at each beam node. They remain the only unknowns in the system of equations. Therefore, the system of equations is much smaller than that of a comparable model with volume elements and the computational effort for solving is reduced considerably. The fact that distortions of the cross section are allowed enables e.g. the calculation of vibrations at frequencies far above the limits of classical theories, while a beam-like modelling is preserved.

Outline
The advantage of the described method is their flexibility. Arbitrary boundary conditions can be applied either on the level of the unit deflection shapes or (with the help of Lagrange multipliers) at any point within the structure. The disadvantage is that - in comparison to hybrid waveguide Finite Element Methods (e.g. [2] or [3]) – a discretisation in lengthwise direction is still carried out.

The aim of this work is to combine the advantages of both. A discretization in lengthwise direction shall be carried out only in nodes of discontinuities due to boundary conditions. In-between these nodes the deflections shall be calculated with the help of wave functions. In order to determine the wavenumbers of these functions one particular method to calculate the unit deflection shapes of the cross section from [1] will be taken: The cross section will be discretized as part of an infinite waveguide and its eigenvalues and eigenvectors will be calculated for distinct frequencies. The eigenvectors are taken as unit deflection shapes for the cross section and the eigenvalues are taken as wavenumbers for the wave function for the lengthwise direction. The real part of the eigenvalue gives a either decaying or growing function while the imaginary part represents an oscillating function. Therefore, an adaption to arbitrary boundary conditions at the points of discretization is possible by a clever combination of those wave functions.

Finally, the method shall be expanded to handle constantly moving loads. By transforming the underlying differential equation form space to wavenumber domain (with the help of integral transform methods) the moving load transforms to a harmonically oscillating stationary load [4]. The result for a moving load should be obtained after solving and back transformation.

References


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