Augmented Beam Elements Using the Semi-Analytical Finite Element Method with Respect to the Theory of Second Order

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In the past decades the Finite Element Method (FEM) became the most popular computational tool in structural mechanics, whereas still beam elements play an important role, e.g. in bridge design. There exist numerous formulations of beam elements which were derived with respect to the assumptions underlying the beam theory (e.g. plane cross-sections or constant shear stress across a section) with the aim of an efficient and accurate calculation. Nevertheless these simplifications are not appropriate for some practical applications.

In this work the beam element solution space is enhanced, so that arbitrary, reasonable deflection shapes of the cross-section can be covered. These unit deflection shapes are calculated by means of the Semi-Analytical Finite Element Method (SAFE Method). It allows to model structures with complex geometric and material properties, for which analytical solutions are not available and is especially appropriate for the numerical computation of structures where these properties do no vary along one direction of primary extent (e.g. the x-direction). The SAFE Method combines the FEM with analytical expressions. The Displacement fields of the cross-sectional plane (y-z-plane) of the Waveguide are described using the FEM while the displacement fields in the wave propagation direction are analytically described by complex exponentials. Thus in comparison to the FEM only the cross-section of the waveguide needs
to be discretized by a two-dimensional Finite Element mesh, which reduces the dimension of the problem and thus saves computational effort compared to a full three-dimensional Finite Element model.

To analyse the stability of beam like structures a further development of the SAFE formulation is necessary, which includes the Theory of Second Order. The geometric non-linearity introduces additional non-linear terms to the strain energy. For analysing small amplitude elastic waves the infinitesimal strain-displacement relations are used, which are linear. But in this case the full strain-displacement relation has to be applied, which finally leads to additional terms in the stiffness matrix of the eigenvalue problem (EVP). By solving this EVP – which depends on the wave number, the angular frequency and the ultimate load – the unit deflection shapes of the cross-section of the waveguide and the ultimate load can be obtained for one prescribed wave number / frequency pair.

The implementation of the Theory of Second Order to the SAFE Method serves for the first order stability analysis (ultimate load analysis) of beam like structures and can be adopted to existing software codes.

References

Kreutz, Johannes (2013): Augmented beam elements using unit deflection shapes together with a finite element discretisation of the cross section. Aachen: Shaker (Schriftenreihe des Lehrstuhls für Baumechanik / Technische Universität München, 10).
